

75. Integration Around a Branch Point

For our final illustration of the use of the residue theorem in evaluating real integrals, we now consider an example involving branch points and branch cuts.

Let x^{-a} , where $x > 0$ and $0 < a < 1$, denote the principal value of the indicated power of x ; that is, x^{-a} is the positive real number $\exp(-a \operatorname{Log} x)$. We shall evaluate the improper real integral

$$(1) \quad \int_0^{\infty} \frac{x^{-a}}{x+1} dx \quad (0 < a < 1)$$