CVEN 3161: Mechanics of Materials I

Laboratory #2: Background and Theory for the Torsion Test

Material Properties: Steel and Cast Iron

In this laboratory session we are going to study the material properties of Steel and Cast Iron as revealed by the Torsion Test. The steel (A36) and cast iron (Gray-40) specimens (**Figure 5**) are solid cylinders 10 in long, which have been machined in the center portion with 6 in of length and a reduced section of 0.75 in diameter. The diameter at each end is 1 in, so the machine may grip and apply a torque to the specimens without introducing effects of stress or strain.

Torsion Definitions

The following are definitions of various torsion material properties that will be determined for both mild steel and cast iron specimens:

(a) Shear Modulus. The shear modulus is analogous to the Modulus of Elasticity. Also known as the modulus of rigidity, this property is represented by G. It is a proportionality constant, which relates shear stress to shear strain. The units of G used in this lab are (psi). In this lab, G is calculated by finding the slope of the linear elastic portion of the shear stress vs. shear strain graph. The reference values of G are 11.5 x 10^6 psi for mild steel and 6.3 to 7.8 x 10^6 for cast iron. You may reference these values in your lab report.

(b) Torsion Yield Stress. The Torsion Yield Stress is analogous to the Tensile Yield Stress. The Torsion Yield Stress corresponds to the maximum shear stress value in the linear-elastic range. On the shear stress vs. shear strain graph, the yield limit will be the point just before the diagram plateaus or levels-off. Similarly in tension or compression, Hooke's Law for shear is applicable for the material up to this point on the shear stress-strain graph.

Theory of Torsion

Torsion stresses and strains behave similarly to shear stresses and strains we have observed in class. The primary assumption of torsion for our purposes is that plane sections remain plane – that is horizontal lines remain horizontal and vertical lines remain vertical during twisting as shown in **Figure 1**.

Shear stress and shear strain can be defined as follows:

$$\tau(r) = \frac{Tr}{I_p} \qquad \text{(Shear Stress)}$$

$$\gamma(r) = \frac{\phi\, r}{L} \qquad \text{(Shear Strain)}$$

where:

$\tau(r)$ = shear stress as a function of the radius, r
T = applied torque (e.g., lbs-in, N-m)
r = radius of the bar
I_p = Polar Moment of Inertia (in^4)
L = Length of the specimen
$\gamma(r)$ = shear strain as a function of the radius
Φ = angle of twist (radians) for pure torsion

The polar moment of inertia is a purely geometrical property. It can be thought of as a ***geometrical resistance*** to twisting. Circular cross-sections are very efficient at resisting torques, unlike square cross-sections, I-beams, or any other shape. The polar moment of inertia for a circular cross-section can be directly computed (Note the units of polar moment of inertia):

$$I_p = \frac{\pi \ r^4}{2}$$

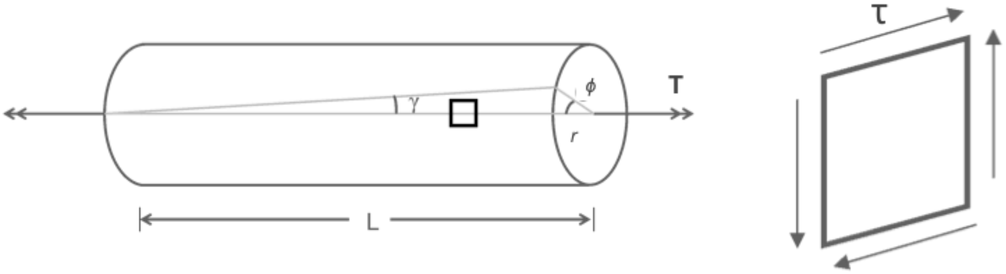

Figure 1. Angle of Twist in Pure Torsion

It is clear from these equations that both the stress and the strain depend on the radius. Thus, they are written as functions of r. The largest shear stress occurs where r has the highest value. The relationship is shown below in **Figure 2a**:

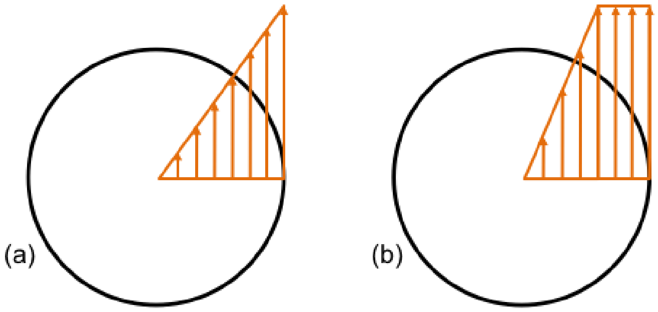

Figure 2. Shear Stress as a Function of Radius in the (a) Linear-Elastic and the (b) Plastic Region.

For an ideally brittle material, the specimen will rupture once the outside fiber reaches the maximum shear stress. This can be seen in **Figure 3** below. For ductile (perfectly plastic) materials, once the stress in the outside fibers reaches the elastic limit, it will stop increasing, and the stress in the internal fibers will start increasing. This is illustrated in **Figure 2b**. The ideal shear stress-shear strain graph for perfectly ductile materials is shown in **Figure 4**.

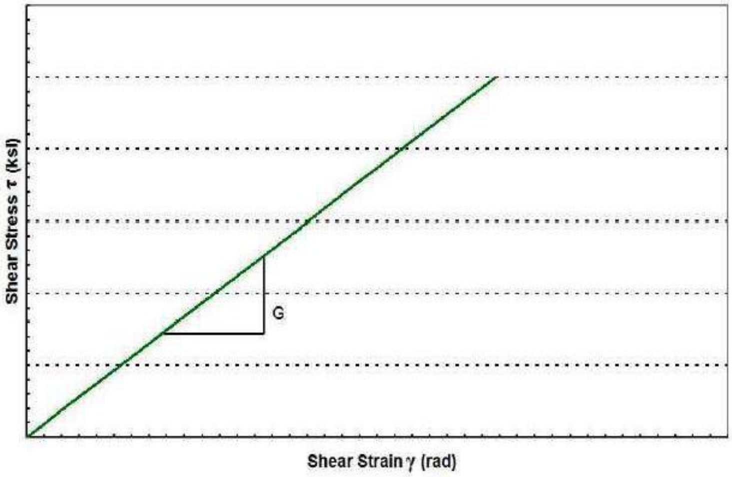

Figure 3. Shear Stress vs. Shear Strain for a Perfectly Brittle Material

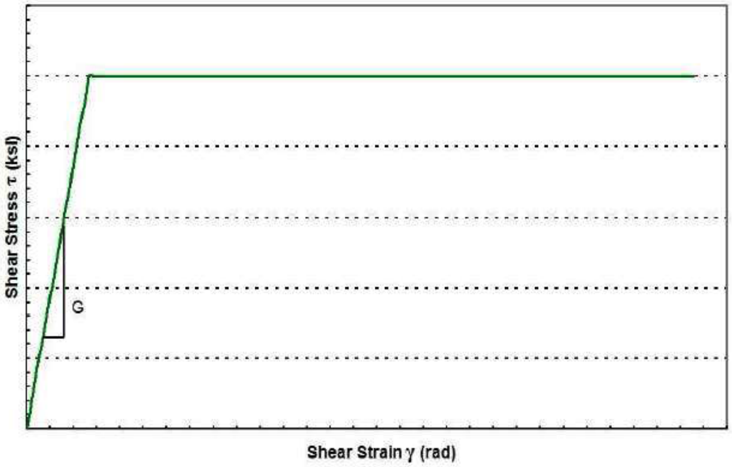

Figure 4. Shear Stress vs. Shear Strain for a Perfectly Ductile Material

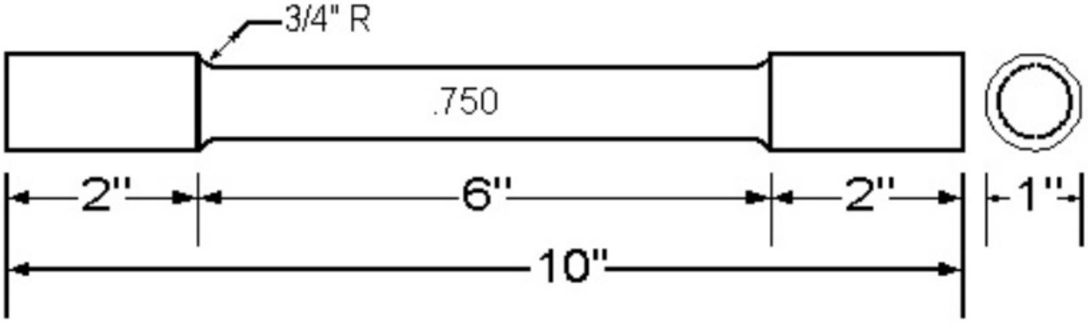

Figure 5. Torsion Specimen dimensions